Rethinking Cartography
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The "Cartographic Program" has investigated interesting, apparently stable, cross-linguistic linear orderings among certain sentence constituents, including left peripheral elements (1a), adverbs (1b) and attributive adjectives (1c).

(1) a. FORCE > TOPIC > FOCUS > TOPIC > FINENESS > TEN... Rizzi (1997)
    b. HABITUAL > REPET > FREQ > VOL > CELERATIVE > ANT... Cinque (1999)
    c. SIZE > LENGTH > HEIGHT > SPEED > DEPTH > WIDTH > ... Scott (2002)

To account for them, cartography's "signature move" is to propose hierarchies of functional projections related by functional selection (2a-c):

(2) a. [FORCE [TOP [FOC [TOP [FIN [TENSE [...]]]]]]]
    b. [HABITUAL [REPETITIVE [FREQ [VOLITION [CELERATIVE [ANTERIOR [...]]]]]]]
    c. [SIZE [LENGTH [HEIGHT [SPEED [DEPTH [WIDTH [...]]]]]]]

Here I consider an alternative approach to the key ordering data. Specifically, I:

- review problematic aspects of functional hierarchies (f-hierarchies).
- contrast linearity as it arises through f-hierarchies and linearity as it arises in mathematical domains (numbers)
- review recent work by Scontras et al 2017a indicating that a single "inequality relation" underlies ordering of attributive adjectives in nominals
- show how this point might be incorporated in a theory of feature-driven projection.

1.0 Functional Hierarchies and Functional Selection

Cartography derives linear orderings by appeal to functional selection: a series of functional heads α, β, γ, δ, etc. in a series of concentric projections αP, βP, γP, δP, etc. where each head stands in its own specific f-selection relation (Σα, Σβ, Σγ, Σδ,...) to the projection beneath it (3):

(3) rΣα1 rΣβ1 rΣγ1 rΣδ1
    [sp α [sp β [sp γ [sp δ [...]]]]] F-selection
    Projection

Relevant linguistic items occupy head or Spec positions. The sequence of f-selection relations determines the sequence of projections determines the order of linguistic items in those projections.

Key property: each head f-selects a unique complement of a unique kind.

This architecture is problematic in various respects.

1.1 The Problem of Explanation
Capturing linear order in a theory ≠ explaining it. Explanation in cartography - why we have the hierarchies we do - must lie in explaining the relevant f-selection relations.

- What is it about a SIZE head (or the size concept) that obliges it to select LENGTHP?
- What is it about a LENGTH head (or the length concept) that obliges it to select HEIGHTP?

Etc.

The f-selection relations are all different, unique to the head type; plausibly the answers to these questions are all different as well.

Cartography offers no answers. Either the sequence of f-selection relations is stipulated as a fact about UG or hope for explanation is directed to semantics.

Semantic selection is based on logical type. This can explain selection between, e.g., D and NP (4a); but not between left peripheral (4b) or modifier (4c) heads & phrases; the latter select and project phrases of the same type (<s,t>, <e,t>).

(4) a. [DP]
    b. [ForceP`s,t>]
    c. [SizeP`s,t>]

Semantics can't help here.

1.2 The Problem of Plenitude
Capturing linear order via local f-selection relations entails that whenever we have two ordered linguistic elements X, Y (say large and broad) in different projections in an f-hierarchy, we must have all projections between them (5a):

(5) a. [SIZEP large [LENGTHP heightP [SPEEDP [DEPTHP [WIDTHP wide [NP board ]]]]]] ✓
    b. [SIZEP large [LENGTHP wide [NP board ]]] ❌!
This is because ordering does not hold pairwise between heads in the hierarchy (5b), but only "by transitivity," through the sequence (5a).

Similarly if the highest projection in a hierarchy is selected by a functional element and the lowest head selects some lexical phrase, the whole hierarchy must be present between, even when no hierarchy elements are realized (6):

(6) [dp the [zep length] height [zep depth] width [np board]]

Broadly put, sentences and phrases must project complete f-hierarchies.

1.3 The Problem of Rigidity
F-selection is not gradable: head a either does or doesn’t f-selects βP. If it does, all elements of αP precede all elements of βP (modulo displacement). F-selection thus yields rigid orders.

In some cases, cartographic hierarchies correspond to rigid ordering judgments (6a). In other cases, they don’t (6b) (Truswell 2009).

(6) a. big red barn ~ *red big barn
b. beautiful big house ~ big beautiful house
c. circular red patch ~ red circular patch

F-hierarchies do not readily accommodate variability or gradability in speaker judgments about acceptable orderings.

2.0 Linear Orders in Mathematics

Numbers, the canonical case of linear order in mathematics, compare interestingly on these issues. Integers (2) exhibit a linear order, displayed in hierarchical fashion via the familiar number line (7).

(7) ... −5 −4 −3 −2 −1 0 1 2 3 4 5 −... 

(i) Each number is related locally to each adjacent number by a relation $\mathcal{R}$. But $\mathcal{R}$ is the same in all cases (i.e., $\prec$)

(ii) $\mathcal{R}$ is not "local"; it holds (or fails to hold) pairwise between all numbers on the line.

(iii) $\mathcal{R}$ can be independently defined. Assuming a prior characterization of positive numbers $\mathbb{P}^+$ and subtraction, $\alpha \mathcal{R} \beta$ iff $\alpha - \beta \in \mathbb{P}^+$ (Beckenbach & Bellman 1961)

Ordering is not stated or explained via the number line (the "numerical hierarchy"). Ordering is defined by relation $\mathcal{R}$ on the domain.

Consequences:

Explanation. We know what explains integer order: the relation $\prec$, which humans cognize and which we can characterize independently of specific numbers. The numerical hierarchy (number line) is entirely derivative on $\prec$.

Plentitude. Since $\prec$ holds (or fails to hold) pairwise between integers, we don’t appeal to "intermediaries" to explain relations between numbers - we don’t say $2 < 5$ in virtue of $2 < 3 < 4 < 5$. Rather $2 < 5$ because $5 - 2 \in \mathbb{P}^+$.

Rigidity. Number ordering is rigid (i.e., $\alpha < \beta \iff \beta < \alpha$ or $\alpha = \beta$) because $\alpha - \beta \in \mathbb{P}^+$ or $\beta - \alpha \in \mathbb{P}^+$ or $\alpha - \beta = \beta - \alpha = 0$. Rigidity is a byproduct of the relation $\prec$ and how it’s defined.

3.0 Subjectivity (Scontras et al 2017a,b)

Scontras et al (2017a) investigated adjectival ordering experimentally. 26 relatively frequent, imageable adjectives from 7 classes (age, color, dimension, material, physical, shape, value). Elicited naturalness judgments on A-A-N object descriptions from 50 participants.

<table>
<thead>
<tr>
<th></th>
<th>Adjective Class</th>
<th>Adjective Class</th>
<th>Noun Class</th>
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<tbody>
<tr>
<td>old</td>
<td>age</td>
<td>good</td>
<td>value</td>
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<tr>
<td>new</td>
<td>age</td>
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<td>tiny</td>
<td>dimension</td>
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<td>purple</td>
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<td>short</td>
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<td>brown</td>
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<td>long</td>
<td>dimension</td>
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<td>wooden</td>
<td>material</td>
<td>smooth</td>
<td>physical</td>
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<td>plastic</td>
<td>material</td>
<td>hard</td>
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<tr>
<td>metal</td>
<td>material</td>
<td>soft</td>
<td>physical</td>
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</tbody>
</table>

Simultaneously, Scontras et al did a corpus study on the Switchboard Corpus and BNC. The preference study and the corpus study were highly correlated (83%).
Scontras et al (2017a) then did 2 follow-up experiments involving **subjectivity** and **faultless disagreement tasks**.

**Faultless Disagreement Task:**
Mary says: "That apple is old,"
Bob says: "That apple is not old".
Rate whether Mary and Bob could both be right, or whether one of them must be wrong.

Scontras et al (2017a): judgments of ordering naturalness and subjectivity as determined by scores on subjectivity judgment & faultless disagreement tasks are highly correlated. Subj explains 85% of variance; FD explains 88%.

"We also see that as the difference in subjectivity approaches zero, the naturalness ratings approach 0.5 (i.e., chance): ordering preferences weaken for adjectives of similar subjectivity (e.g., "yellow square" or "fresh soft"). p.58

Scontras et al (2017a) expanded their experiment to a larger class of adjectives (78) and a larger subject pool to rate naturalness (495). They also recruited a larger pool of subjects (198) to rate subjectivity & faultless disagreement. The results were basically the same.

Scontras et al., 2017b investigated **adjective inherentness** (how essential an A’s meaning is to N it modifies), **intersective vs. subective modification** (the mode by which A composes with the N it modifies), and **concept formability** (whether A composes with N to form a complex, idiomatic concept). In all cases, subjectivity was a better predictor of order.

The results of Scontras et al 2017a,b suggest something like the "mathematical picture" for adjectival ordering, viz.,

- there is an "inequality" relation that speakers can judge between APs: \( \leq_{\text{subj}} \)
- \( \leq_{\text{subj}} \) induces an order on the domain of AP property classes (size, length, etc.)
- grammar appears to make use of \( \leq_{\text{subj}} \) in syntactic projection.

**Attractions:**

**Explanation.** We would know what explains AP order: the relation \( \leq_{\text{subj}} \), which humans cognize and which we can characterize independently of specific As. The f-hierarchy would be derivative on \( \leq_{\text{subj}} \).

**Plenitude.** Since \( \leq_{\text{subj}} \) holds pairwise between APs, we don’t appeal to ”intermediaries” to explain order - we don’t say large precedes wide because Size precedes Length precedes Height precedes Speed precedes Depth precedes Width. Rather large precedes wide because Size \( \leq_{\text{subj}} \) Width.

**Rigidity.** Adjectival ordering is predicted to be as rigid as \( \leq_{\text{subj}} \). For adjectives A1, A2 for which A1 \( \approx_{\text{subj}} \) A2, we predict fluidity.

**Question:** Can one incorporate these results into cartography via f-selection?

**Answer:** It’s hard to see how. Consider P:

P: X selects YP if Y is lower wrt \( \leq_{\text{subj}} \).

"Selection" here can’t be f-selection (it’s not category specific). P is just a restatement of the facts. Can we implement something like the "mathematical view"?

**4.0 Projection from Ordered Feature-Sets**

Cartography "explodes" traditional heads like CP and AP (8a,b). This move yields no way of ordering those heads via a single relation.

\[(8)\]

\[C \overset{\gamma}{\preceq} N\]

\[AP \overset{\gamma}{\preceq} \{\text{FORCE, TOP, FOC, TOP, TRENSE […]}, \text{SIZE, LENGTH, HEIGHT, SPEED, DEPTH, WIDTH […]}\}\]

**Idea:** (i) Recast f-projections as features born by unexplored heads H.

(ii) Order features on H by relation \( \preceq \)

(ii) Align projection order with feature order via agreement.

\[(9)\]

| a. | HP | \[F_1]\{[F_1],[F_2],[F_3]\} | \[E_1]\ | \[E_2]\ |
|----|----|-----------------|---------|
| b. | HP | \[F_1]\{[F_1],[F_2],[F_3]\} | \[E_1]\ | \[E_2]\ |
| c. | HP | \[F_1]\{[F_1],[F_2],[F_3]\} | \[E_1]\ | \[E_2]\ |

**ORDER:** \[F_1 \overset{\alpha}{\preceq} F_2 \overset{\beta}{\preceq} F_3\]

**ALIGNMENT:** Agree from lowest ranked to highest ranked feature.
There is a Minimality Problem in (9b): \([F2]\) agrees w/H across a closer \([F1]\) of the same type; similarly for (9c). This can be resolved by interpolating "light heads" (h):

\[
\begin{align*}
(10) & \quad & \text{(a)} & \quad \gamma & \quad \overrightarrow{H} & \quad \text{[F1],[F2],[F3]} & & \quad \text{(b)} & \quad \gamma & \quad \overrightarrow{H} & \quad \text{[F1],[F2],[F3]} \\
& & \quad \text{(c)} & \quad \gamma & \quad \overrightarrow{H} & \quad \text{[F1],[F2],[F3]} & & \quad \text{(d)} & \quad \gamma & \quad \overrightarrow{H} & \quad \text{[F1],[F2],[F3]}
\end{align*}
\]

This picture has a near-perfect execution within Pesetsky and Torrego 2007, which postulates a three-way division in features:

\[
\begin{align*}
(11) & \quad & \text{(a)} & \quad \text{iF} & \quad \text{interpretable F, associated with a "meaning"} & & \quad \text{(b)} & \quad \text{Fval} & \quad \text{valued F, associated with visible marking/pronunciation} & & \quad \text{(c)} & \quad \text{F} & \quad \text{uninterpretable-unvalued F, concordial}
\end{align*}
\]

Let F's be

a. interpretable on "arguments" of H
b. valued on H and h (@ one valuation per head)
c. uninterpretable-unvalued elsewhere

\[
\begin{align*}
(12) & \quad & \text{(a)} & \quad \overrightarrow{HP} & \quad \text{[F1],[F2]} & & \quad \text{(b)} & \quad \gamma & \quad \overrightarrow{H} & \quad \text{[F2val]} & & \quad \text{(c)} & \quad \gamma & \quad \overrightarrow{H} & \quad \text{[F2val]} & & \quad \text{(d)} & \quad \gamma & \quad \overrightarrow{H} & \quad \text{[F2val]}
\end{align*}
\]

Repeat this sequence of operations for \(\text{small}\) and the \textit{Pro} subject of \(d\text{P}\).

\[
\begin{align*}
(14) & \quad & \text{Pro} & \quad \overrightarrow{dP} & \quad \text{[F1]} & & \quad \text{d[SCOPE]} & \quad \overrightarrow{dP} & \quad \text{[F1]} & & \quad \text{d[DIMENSION]} & \quad \overrightarrow{dP} & \quad \text{[F1]}
\end{align*}
\]

4.1 Projecting Adjectival Modifiers (Cinque 2010, Scott '02, Laezlinger '05)

- Recast cartographic f-hierarchy as \([\text{MAT}],[\text{COL}],[\text{SHP}],[\text{PHYS}],[\text{AGE}],[\text{VAL}],[\text{DIM}]\,...\)
- Replace cartographic f-heads with the single head (e.g., D/d)
- Order by \(\leq\) yielding \([\text{MAT}] \leq [\text{COL}] \leq [\text{SHP}] \leq [\text{PHYS}] \leq [\text{AGE}] \leq [\text{VAL}] \leq [\text{DIM}]\)...
4.3 Projecting the Left-Periphery (Rizzi 1997)

- Recast cartographic f-hierarchy as \{\text{FIN}, [\text{TOP}2], [\text{FOC}], [\text{TOP}1], [\text{FOR}]\}
- Replace cartographic f-heads with the single head \(e\) (e.g., \(E/e\) from Banfield 1973)
- Seek an ordering relation \(\mathcal{R}\) yielding \([\text{FIN}] \leq \_ \_ \_ [\text{TOP}2] \leq \_ \_ \_ [\text{FOC}] \leq \_ \_ \_ [\text{TOP}1] \leq \_ \_ \_ [\text{FOR}]\)

\[
\begin{align*}
\text{why} & \quad \text{eP} & \quad \text{e}' & \quad \text{EP} & \quad \text{TP} \quad \text{[\text{FIN}]} \\
\text{who} & \quad \text{eP} & \quad \text{e}' & \quad \text{EP} & \quad \text{TP} \quad \text{[\text{FIN}]} \\
\end{align*}
\]

4.3 Rethinking the Cartographic Project

- Recast all cartographic domains not predictable by semantic type as feature sets
- Replace cartographic f-heads with a single head \(H/h\) relevant to the domain
- Analyze \(H\) as bearing subsets of feature from the domain
- Seek a single ordering relation for the set (i.e., generalize Scontras et al).

5.0 Conclusion

Proceeding this way would analogize the cartographic project in syntax to the most successful cartographic project yet executed in linguistics, viz.: universal phonetics. Consider the familiar “cartography of human vowels”.

Vowel space is determined by extra-linguistic anatomy/gesture/acoustics. The linguistic system digitizes this space with features, identifying perceptually salient, acoustically stable, gesturally replicable feature-bundles as segments. Feature-relations (front/central/back) reflect extra-linguistic organization.

There seems to be an extralinguistic (cognitive) space of attributes associated with objects. The linguistic system digitizes this with features, identifying stable bundles as modifier concepts. Feature-relations (ordering) reflect extra-linguistic organization (subjectivity).

REFERENCES