SUPERSYMMETRY AND SUPERGRAVITY (Fall 2024)

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Supersymmetry (susy) and supergravity (sugra) were discovered in the 1970's (sugra even here at Stony Brook. The 50th anniversary will be celebrated in 2026, here and elsewhere). They play a major role in modern theoretical physics, but they are usually briefly introduced and then used as essential parts of string theory. Here we shall not assume any familiarity with susy or sugra, and give much more background material to derive and explain their structure. We do not discuss, nor assume any familiarity with string theory. I last taught this class in 2015(!). "For posterity" (I am told) it will be filmed. For that reason the lectures will take place in the Simons Center. The time and room will be determined in consultation with students. Typed notes will be handed out in class.

Prerequisites:

1) **Classical (not quantum) field theory:** Dirac, Yang-Mills, and Klein-Gordon actions in $d = 4$ dimensions; Dirac matrices $\{\gamma^m, \gamma^n\} = 2\eta^{mn}$ in n dimensions (will be reviewed); Euler-Lagrange field equations; Noether theorem (will be reviewed). We use two-component spinor formalism in $d = 4$ (will be reviewed).

2) **Group theory:** Basics of Lie algebras (not yet superalgebras): finite-dimensional representations of $SU(N)$ and $SO(N)$ (including spinorial representations).

3) **General Relativity:** Tensor calculus, Hilbert-Einstein action, its general coordinate invariance. (We shall use the tetrad (vielbein) formalism, rather than the metric formulation, but this will be introduced, and is not assumed to be known).

Since this is an advanced class, I will dispense with the usual oral and final exams, but students are expected to do the homeworks, and give at the end a short (15 minutes) presentation about one of the topics of the class. The grading is pass/fail.

Contents (a choice of topics will be made in consultation with students):

1) We begin with susy and sugra in quantum mechanics(!). Many of the central ideas (closure of the gauge algebra, emergence of gravity, constraints in superspace) are already present here. This part of the class is very easy to follow, and real fun.

2) Then we discuss susy in $d = 4$: the WZ model, the $N = 1$ susy YM theory, (super)conformal symmetry of these models, super-QED, the $N = 2$ models, and the $N = 4$ model. The latter will be rederived from Kaluza-Klein (KK) dimensional reduction from 10 to 4 dimensions. Spontaneous susy breaking and Goldstinos. Loop calculations and finiteness. Susy nonlinear sigma models.

3) Next comes sugra: the $N = 1$ $d = 4$ Poincaré model (its local susy invariance is discussed in detail). Next the anti-de Sitter sugras. Then the $d = 11$ and the three $d = 10$ sugra models (the $N = IIA, IIB$ and $N = 1$ sugra). KK reduction of the IIB model to the $N = 8$ gauged $d = 5$ sugra (the basis of the AdS/CFT relation in string theory).

4) Gauging of (super)algebras. $d = 2$ sugra (the starting point of superstring theory). Induced representations. Conformal sugra in $d = 4$. Nonlinear realizations of susy.

5) Superspace: we introduce the general coset approach, with Lie derivatives, vielbeins, and covariant Lie derivatives and covariant derivatives, supertorsions and supercurvatures, and Haar measure for integration. Then we apply it to rigid $N = 1$ susy. Quantization in superspace and loop calculations. We construct the minimally supersymmetric Standard Model (MSSM). $N = 2$ and Seiberg-Witten theory. Finally, we discuss the superspace approach to sugra, with constraints on the geometry (supervielbeins and superconnections), Bianchi identities, and prepotentials.