

PHY680.02: Group Theory

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Group Theory is the theory of symmetries. Since symmetries have become central in modern physics, group theory is widely used: in atomic and molecular, solid state, nuclear, and particle physics. In this class we first introduce the concepts of group theory by studying finite groups, but then we focus on Lie algebras and Lie groups.

Prerequisites: There are no mathematical or physical prerequisites for this class; the course is just fun and accessible to any graduate student. For some of the applications, a very small amount of classical field theory (the coupling of Yang-Mills fields to Dirac fields) is useful, but this material will be provided in class.

Grade: There will be homework each week, a midterm exam and a final exam (consisting of both a written and an oral part). The grade will be calculated according to $1/3 + 1/6 + 1/2$.

Literature: For finite group theory we recommend W. Ledermann, "Introduction to the Theory of Finite Groups". This is a clear and very readable introduction at the undergraduate level. Most of the class deals with Lie algebras and Lie groups. Here we use the classic H. Georgi, "Lie Algebras in Particle Physics". One a slightly more advanced level, both for finite and for Lie groups, we recommend P. Ramond, "Group Theory, a Physicist's Survey". These (and other) books are on reserve in the Math/Physics library. In addition, we hand out copious notes.

Office hours: Appointments can be made after class or by email.

Topics: We have chosen those subjects which we have found to be essential for graduate students. We begin with an introduction to finite group theory, where we define various concepts such as invariant (normal) subgroups, classes and cosets. Then we discuss two physical applications: (1) spinors in arbitrary Euclidean and Minkowskian dimensions, and their reality properties, and (2) the spectrum of small molecular vibrations of polyhedra and buckyballs. Next we turn to Lie algebras. First we define the classical groups $SU(N)$, $SO(2N)$, $SO(2N-1)$, and $USp(2N)$, we introduce roots, fundamental weights, conjugacy classes, Young tableaux, Dynkin diagrams, and construct their irreducible representations (irreps). Next we give a simple construction of the exceptional Lie algebras G_2 , F_4 , E_6 , E_7 , E_8 . Finally we present some material that is much used in physics: the coset construction and covariant Lie derivatives, the Haar measure for group and coset integration, non semi-simple group theory, unitary irreps for non-compact simple groups, such as the Lorentz group, and Wigner's construction of unitary irreps for non-semisimple groups such as the Poincaré group.

We end with some interesting generalizations: Kac-Moody algebras and superalgebras.