Course Outline for Advanced Quantum Field Theory

This class is for graduate students who have followed a class in ordinary quantum field theory. For example, students are assumed to be able to compute the divergent and finite parts of one-loop Feynman diagrams in QCD with ghosts using ordinary dimensional regularization, use path integrals to derive Feynman rules, and be familiar with the one-loop β functions of the renormalization group.

Typed notes will be distributed. There are weekly homeworks, a midterm exam and two final exams (one written exam, one oral exam). The topics to be discussed are selected from the following list (and can be chosen by the registered students to some degree).

Instantons. Selfdual solutions and winding number, regular and singular solutions, zero modes and moduli (bosonic and fermionic), two-component spinors in Euclidean space, index theorems for moduli, (super)conformal algebra and induced representations, zero modes and conformal symmetries, the measure for bosonic and fermionic collective coordinates. Large instantons and the Higgs effect, tunnelling and θ-vacua, the strong CP problem, false vacua and phase transitions, the $U(1)$ problem, baryon decay, instantons at finite temperature.

Anomalies. $V - A$ and chiral basis; triangle, box, and pentagon anomaly. Gauge anomalies ruin unitarity and renormalizability, group theory of anomalies. Consistent and covariant anomalies, the Wess-Zumino integrated anomaly, descent equations. Pauli-Villars and Fujikawa and point splitting regularization; anomalies using dimensional regularization by dimensional reduction, gravitational γ_5 anomaly.

On-shell renormalization of QED (needed for 2-loop $g-2$). One-loop calculation of photon selfenergy, fermion selfenergy and vertex correction: UV and IR divergences, cancellation of IR divergences in amplitudes at $Q = 0$. Cancellation of IR divergences in cross sections of QED. Differences with QCD. IR divergences in massless $e^+e^- \to \mu^+\mu^-$ scattering, collinear divergences.

Theory of the $g-2$ experiment. History of the normal and anomalous magnetic moment. One loop result. Two loop mass-independent and mass-dependent corrections. Calculation of the hadronic vacuum polarization (HVP) contributions to a_μ. Higher-loop corrections. Present status of a_e and a_μ.

Solitons. Classical solutions: kinks; vortices; Dirac, Wu-Yang and 't Hooft-Polyakov monopoles. One-loop corrections to mass and (in susy theories) central charges. Index theorem for spectral densities.

Dirac formalism, Hamiltonian path integrals, BRST symmetry, BV formalism. Primary and secondary constraints; first class and second class constraints, structure functions, the Hamiltonian BRST charge Q_H, the BRST invariant Hamiltonian, the quantum action, boundary conditions and gauge-choice independence. Antibrackets, the BRST charge is the action, reducible gauge theories and ghost-for-ghosts, the master equation. Examples.

Background field method. Background gauge invariant quantum action, renormalization of background gauge field theories, background-gauge invariant effective action, calculation of the β function of QCD at two loops, gauge parameter independence of the β function.

Renormalization of unbroken nonabelian gauge theories. Ward identities for divergences in proper graphs, proof of multiplicative renormalizability to all orders, nonlinear and noncovariant gauges, asymptotic freedom in the Coulomb gauge.

Renormalization of Higgs models. Renormalization of Goldstone models, the Goldstone theorem at one- and two-loop level, the spontaneously broken $SU(2)$ model, renormalization of the $SU(2)$ Higgs model.

Path integrals in QM. Path integrals for the anharmonic oscillator, boundary conditions, one- and two-loop corrections to the ground state and first excited state from either QM perturbation theory or path integrals, comparison with the WKB method, tunnelling in QM, multi-instanton corrections to tunnelling, Green functions in a kink background. Susy and Sugra in QM, the fermionic anharmonic oscillator and susy. Weyl ordering for bosons and fermions. Nonlinear susy sigma models (NSSM) in superspace, NSSM for internal symmetries. Path integrals for nonlinear sigma models. Time-slicing, mode regularization, and dimensional regularization on a finite interval. Gravitational anomalies in two dimensions from QM. Chiral anomalies and Lorentz anomalies in higher dimensions from QM. Cancellation of anomalies in 10-dimensional supergravity theories. Trace anomalies from ordinary and susy QM.

Unitarity. The largest-time equation, unitarity for scalars from the cutting rules, unitarity for spin $\frac{1}{2}$ (similar to the spin 0 case), unitarity for unbroken gauge theories with Ward identities, unitarity for spontaneously broken gauge theories with Ward identities, the interplay between unitarity and renormalization, locality and causality and statistics.