

Applied Mathematics and Statistics
Foundation Qualifying Examination Part B
in Computational Applied Mathematics
Spring 2021 (January)

Date of Exam: January 25, 2021, **Time:** 11:15 AM – 13:15 PM

Instructions: There are 3 problems, and you are required to solve all of them. All problems are weighted equally. Please show detailed work for full credit.

- This exam is conducted via Zoom on January 25 from 11:15 am to 1:15 pm EST.
- The entire Zoom meeting and chat messages are being recorded.
- This is a closed book, closed note exam.
- Hand calculators (or other computing devices) may not be used during the exam.
- You should join the Zoom meeting from two devices: Your computer/laptop/tablet (with webcam), and your smartphone (with camera).
- Audio should be muted and video must be kept on during the exam.
- Your computer webcam must fully show your face; your smartphone camera should show your computer monitor, your hands and workspace, with the pages of paper being used for the exam.
- At the very beginning of the exam, during set up, you will be asked to do a brief “environment scan”, showing the workspace where your computer is and the desk/table/floor where you will be writing your work.
- You are required to bring enough blank pieces of paper to use for the exam. You will show the blank pages at the beginning, during the “environment scan” on Zoom.
- You are not allowed to use the internet for any searching or communication with others, with the sole exception of communication privately with the proctors via Zoom chat (which is set so that your chats only go privately to hosts, not to others).
- After you finish the exam, scan your pages, numbered, ordered and oriented appropriately, into a single pdf file. Email the pdf file to **roman.samulyak@stonybrook.edu** no later than 5 minutes after completion of the exam (i.e., **by 1:20 pm EST**).
- No students are allowed to leave the Zoom meeting until the exam is over.
 - If you finish the exam early, then submit your exam and remain in the Zoom meeting until the conclusion of the exam at 1:15 pm EST.
 - After submitting your exam, you can study for another exam or work on anything else, while staying in view in the Zoom meeting.
- If the answers are not submitted by 1:20 pm EST, the exam will not be graded, and a score of zero will be given.
- If you have a question during the exam, then send a chat message to the host privately.

B1. Consider the following initial value problem with a small positive parameter ε

$$y'' + (2 - \varepsilon)y' - 2\varepsilon y = 0, \quad x \in [0, +\infty),$$

$$y(0) = 1, \quad y'(0) = \varepsilon.$$

- a) (2 points) Find the exact solution.
- b) (5 points) Obtain the 0-order and the 1st order perturbation series solutions.
- c) (3 points) Compare both the 0-order and the 1st order perturbation solutions with the exact solution and explain your results.

B2. Given a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, where $m \geq n$. Assume \mathbf{A} has full rank, and \mathbf{A} is composed of a subset of rows of \mathbf{B} . Let $\mathbf{A} = \mathbf{Q}_1 \mathbf{R}_1$ and $\mathbf{B} = \mathbf{Q}_2 \mathbf{R}_2$ be the reduced QR factorization of \mathbf{A} and \mathbf{B} , respectively.

- a) (4 points) Show that $\|\mathbf{A}\|_p \leq \|\mathbf{B}\|_p$ for any $p \in [1, \infty]$.
- b) (2 points) Show that $\|\mathbf{R}_1\|_2 \leq \|\mathbf{R}_2\|_2$.
- c) (4 points) Show that $\|\mathbf{R}_1^{-1}\|_2 \geq \|\mathbf{R}_2^{-1}\|_2$.

B3. Given $\mathbf{A} \in \mathbb{R}^{n \times n}$ and a unit length vector $\mathbf{q} \in \mathbb{R}^n$.

- a) (5 points) Show that there exists an orthogonal matrix $\mathbf{Q} \in \mathbb{R}^{n \times n}$ whose **first** column is \mathbf{q} , such that $\mathbf{Q}^T \mathbf{A} \mathbf{Q}$ is an **upper** Hessenberg matrix.
- b) (5 points) Show that there exists an orthogonal matrix $\mathbf{Q} \in \mathbb{R}^{n \times n}$ whose **last** column is \mathbf{q} , such that $\mathbf{Q}^T \mathbf{A} \mathbf{Q}$ is an **upper** Hessenberg matrix.